THE PRESIDENT'S PRIZE ESSAY

By Gordon Petrie

Subject

Review the present position of control by photogrammetric methods, and discuss in the light of possible developments in apparatus and techniques the extent to which you think ground traverse and triangulation can be superseded.

The essay was submitted under the pseudonym *Sahib an Nuksha.*

Introduction

Even the most casual reader of photogrammetric literature can hardly fail to be struck by the enormous interest in techniques and instruments which will provide control by photogrammetric methods. It would appear that present methods of plotting detail from photographs are fairly well established, and meet the demands of most users. The provision of control is however still a burning question and a flood of papers appears advocating one system or another, giving new methods of computation and adjustment or giving details of new and better instruments. A review of present methods of providing control and the impact of possible new techniques and instruments is therefore topical and instructive.

The economy of photogrammetric mapping is very much governed by the extent to which the necessary control points have to be provided on the ground. The less ground control needed, the greater the possibilities for producing photogrammetric maps quickly and economically. Provision of control directly from photographs can be done using either terrestrial or aerial methods, but only the latter is of importance to-day. It is normally carried out either as radial triangulation or as spatial aerial triangulation.

Radial Triangulation

The basic principles of radial triangulation are well established both in theory and in practice and need no repetition. At present graphical radial methods (e.g. the Arundel and Hand Templet methods) are little used, their place having been taken by mechanical radial triangulation using slotted templets. Even this method, still used for small-scale mapping in a number of offices, must be considered to be on the decline. The advantages of having no instrumental measurements to make, no computation and of carrying out the work using relatively unskilled personnel do not in general make up for inadequate accuracy for many jobs and the fact that only planimetric coordinates can be obtained from the method.

Fagerholm[1] has shown that the accuracy of the Slotted Templet method can be improved by placing small ball bearings between the templets and by vibrating the base board tilted in each of four directions, pricking through the points, and accepting the centre of gravity of the four positions so fixed. This rather complicated procedure has not so far achieved much favour.

The use of stereo-templets is another method of improving the accuracy of radial triangulation using the orthogonal projection of a stereo-model in a plotting machine rather than the perspective projection of a single photo. It is a method which has come into extensive use in at least two very large organisations—the U.S. Geological Survey and the French Institut Géographique National. The need to use a plotting machine takes away much of the attractive simplicity of the slotted templet method, but the increase in accuracy is worthwhile and there is the additional advantage that triangulation for height can be carried out simultaneously in the plotting machine. The Institut Géographique National at the Stockholm Congress gave the results of a large project in Africa requiring 1/50,000 mapping where only sparse ground control existed, mainly astro fixes, with extra heights produced by barometric levelling. One hundred and sixty-eight photos in 7 parallel strips and 3 transverse strips were used, and the results of the stereo-templet triangulation, to provide the extra control needed for plotting, based on 8 perimeter points, gave a mean square error in X and Y at the check points of 0·3 mm. at the negative scale. The simultaneous triangulation for height gave a mean square error of ± 6 metres based on 9 points of barometer control, a figure reduced to ± 4 metres when based on 24 height control points.

A solution of this accuracy is quite adequate for most medium to small scale mapping and no doubt the block adjustment for planimetry given by the stereo-templet procedure combined with the Institut Géographique National mechanical block adjustment for height, which cut out much tedious time-consuming and expensive computation, make it even more attractive and economic from the Institut Géographique National's point of view. More organisations can be expected to use this solution for small scale mapping in the future.

Another approach in radial triangulation is the analytical method used in Germany, Belgium and particularly Holland in the 1930's. Measurement of direction on the photographs was made in a radial triangulator and the coordinates of the wing points computed. Adjustment of the scale and azimuth of a strip was made by comparison of the computed against the terrain coordinates of control points in the first and last models. This method has fallen almost wholly into disuse due to the very considerable computation involved, the inaccuracies in angular measurement due to tilt and relief differences, and the lack of coordinates in height.

Currently there is some revival of interest in the method largely due to the introduction of high speed electronic computers which should ease the computation burden considerably. Professor Roelofs[2] has shown that the systematic errors in scale and azimuth transfer through a strip in radial triangulation are very small compared with those encountered in spatial aerial triangulation with the small tilts encountered in modern photography. He has also shown how radial triangulation is suited particularly to super-wide-angle photography as the ground inclination of a radial line is smaller as the ground length of the line increases, which will be the case with super-wide-angle photos. The introduction of super-wide-angle photography will also be beneficial as it will cut down the number of rhomboids needed to cover a certain distance.

The adoption of equipment such as horizon cameras and the solar periscope to record the position of the nadir point would be a further help to the adoption of analytical radial triangulation. As the new Wild RT-1 radial triangulator measures not only the directions of points on the photographs, but gives the full polar coordinates (distance and direction) of any point measured, one assumes that heights could be computed in addition to the normal planimetric coordinates in the same way as is done with the rectangular coordinates measured by a stereo-comparator.

Generally however it seems that for the immediate future we will only see radial triangulation applied for small scale mapping with more use of stereo-templets and less slotted templet work.

Spatial Aerial Triangulation

This method is by far the most important one for providing control by photogrammetric means. The first real use of the method came with the introduction of the Multiplex in the middle 1930's. Multiplex bridging was greatly used during the 1939–45 war and in the years immediately after, particularly for medium and small scale mapping, but it has largely been superseded to-day by triangulation in first-order machines. These machines with their much higher accuracy and flexibility allowed the use of greater flying heights which resulted in economies both in the number of photographs and the amount of control required to map a given area at a certain scale.

Only a few Multiplex were ever made with 20 or more projectors and by far the majority of these instruments are equipped with 6 or 7 projectors. This places a limit on the method also, the solution being either to bridge a strip in pieces which results in discontinuities or to provide ground control every sixth pair, which is a very uneconomic solution for small scale work. With long strips, too, the graphical solution of the Multiplex results in great lengths of paper which are extremely difficult to handle, even more so if there are several of these strips which must be combined for block adjustment.

Even with these disadvantages and the replacement of Multiplex bridging by triangulation with first-order machines, the fact that large numbers of Multiplex still exist in many organisations means that bridging will still be used even if it is not the best and most economic method of providing control. Recent developments have in fact overcome some of the disadvantages outlined above, particularly the introduction of stereo-templets. Having individual sets of templets for each pair or for two or three pairs gets over the handling problem, and allows block adjustment of several strips in an easy way. It allows too the use of a 6-projector bar without having full ground control every sixth model, only heights being needed at each end. Another development is the use of 2 projector Multiplex units with inclinometers for bridging long strips, a method which would again be suited to the use of the stereo-templet.

In general, however, these developments merely give some extra life to the method of Multiplex bridging which seems destined to be replaced due to the higher precision and therefore greater economies which can be obtained by other spatial triangulation instruments and methods.

Currently the most accurate economic, and a popular method of providing control is to use first-order machines such as the Wild A7, Zeiss Stereoplanigraph C8 and Santoni Stereo-cartograph IV which possess the full parallelogram of Zeiss. The normal aeropolygon method used with these machines is similar in principle to Multiplex bridging and needs no explanation. The very high precision of these instruments together with the excellent qualities of modern photography have resulted in very accurate results, which allow the application of extensive aerial triangulation to all mapping work except at very large scales, where the demands for precision, particularly in height, are such that they can as yet only be met by ground survey.

The accuracy achieved by these first-order machines has allowed the use of greater numbers of photographs between ground control points, the actual number being decided by the final precision demanded. The economy in control from use of greater flying heights has already been mentioned, and a further economy in time and money in the aerial triangulation process itself has been brought about by the introduction of electric coordinate recording, which gives directly strip coordinates ready for computation and adjustment, if necessary, in a form suitable for electronic computers.

It is not often realised how much ground control is saved even by using a photogrammetric plotting machine on a single overlap with 4 to 6 ground control points, particularly at large scales. Colonel Irwin[8] of the Ordnance Survey has given the figures of 80–90 minor control points per sq. km. necessary for 1/1250 mapping by chain survey, which dropped straight away to 12 to 14 control points per sq. km. when 1/1250 survey was undertaken by the Wild A5 using 1/3500 scale photography. This figure will be cut down considerably if greater flying heights are used or if aerial triangulation can be satisfactorily applied at this scale, as perhaps may have been done since Colonel Irwin's paper in 1955.

One of the reasons for continuing to supply full ground control at large scales, and even many extra points, has been the desire to retain a network of control points for continuous revision work which has to be carried out on the ground later. It now seems from the results published by the Bundesamt für Eich- und Vermessungswesen in Vienna that the precision of coordinates derived from photogrammetric coordinates measured in the Wild A7 is as good as those derived from normal traversing on the ground with a theodolite and tape. Using 1/9000 photography taken with a Wild RC7 glass plate camera, this organisation has compared the coordinates of signalised points fixed by traverse every 300 metres with those obtained by registering the machine coordinates on the A7. The average absolute closing error in the direction of the traverse was 6·2 cm., and at right angles to the traverse line 4·9 cm. The result is that no further traversing will be carried out on the ground and the coordinates of the revision points will only be determined from the machine.

The economy of such a procedure has always been apparent but the question of precision has been more doubtful. The Austrian figures show that this has been overcome largely by the creation of very favourable circumstances in the use of ultra-flat glass plates in the aerial cameras, compensating plates in the machine, signalised points and air-conditioned rooms for the equipment. It would seem that in Europe at least, where the demand for large scale cadastral and topographic maps and the desire for continuous revision are greatest, such a method will be used increasingly in the future. The self-registering devices of the Zeiss C8 and Wild A7 allow the work on the machine to be speeded up enormously, again resulting in economies for the photogrammetric method as against normal traversing on the ground. As the method can also be carried out simultaneously with graphical plotting of a map on the plotting table, it has even more to commend it.

The self-registering devices of the A7 can also be used with the Wild A8 which can therefore also be employed economically for numerical work such as the coordinating of cadastral boundary marks and revision of control points in a single model, an important point when one sees the number of A8's that have been made. The other two instruments, the Santoni Simplex III and Thompson-Watts Plotter, which can rank with the A8 as first-order *plotters*, have not yet appeared with electric registration, and are therefore unlikely to be economically used in co-ordinating cadastral and revision points.

It is of course possible to use these three instruments for the provision of further control by aerial triangulation, but as they are not designed to do so with the same facility and economy as a full first-order machine such as the Wild A7, they are unlikely to be used for such work except in the event of the first-order machine not being available.

By far the greatest talking point to-day with regard to the future supply of control by photogrammetric methods is the question of applying analytical aerial triangulation, a method long known, but one which only now can be applied economically with the development of electronic computers. The flood of papers referred to in the very first paragraph of this discussion is largely devoted to this exciting new technique. Most of these papers deal with the mathematical side—the different ways of forming a model, of joining models into strips and strips into blocks, of adjusting individual strips and blocks to ground control using least-square methods, of applying precise compensations for known systematic errors and so on. The more average photogrammetrist scanning such articles can only feel awe at the authors' abilities to master the complicated nomenclature and the endless series of complex equations required for the computations and mentally place them among T. W. Williamson's "mathematical gymnasts and research boffins".[4] For all that, it is quite apparent that the analytical method is one which offers a number of advantages over the methods of aerial triangulation generally applied to-day, and is of great importance for the future.

Without attempting an exhaustive list, the chief advantages of the analytical method would appear to be as follows:—

(*a*) The application of very precise compensation during computation of systematic effects such as lens distortion, film shrinkage, earth's curvature and refraction which affect the precision of current aerial triangulation with first-order machines.

(*b*) The measurement of photo coordinates using a stereocomparator can be made much more accurately than with a first-order machine with its larger systematic and accidental errors in measuring model coordinates.

(*c*) The possibility of utilising a less complex and therefore cheaper instrument in the stereocomparator rather than the first-order machine.

(*d*) The possibility of simultaneously solving and adjusting by least squares a complete strip or block of photographs to all given ground control as is done with blocks of conventional ground triangulation.

(*e*) The advantage of using auxiliary control data obtained by statoscope, radar altimeter, horizon cameras, gyroscopes, shoran etc. in the computation.

(*f*) The possibility of complete automation of the whole aerial triangulation procedure with resultant economies in time and money.

These are formidable advantages—particulary the theoretical higher accuracies which can be expected from more precise measurements and the rigorous, correct treatment of all possible information. The questions of organising the work efficiently and the determination of the systematic errors listed in (*a*) above which might be troublesome in reaching this theoretical accuracy can surely be overcome.

More serious however is the fact that, in spite of the great research and work put into analytical aerial triangulation, there has been an almost complete absence of practical results to show that these theoretical advantages in accuracy and economy in proving control have been realised. The only published results of any extensive work have been those of the Ordnance Survey[3, 5] between 1953–5 on various experimental blocks at different scales. In general these gave results comparable in accuracy but not in speed with those obtained by conventional first-order machines in other organisations. This work was of course undertaken with the old Cambridge stereocomparators which have neither the accuracy nor the self-recording devices of the currently produced instruments.

It is of importance therefore to look into the possible practical difficulties that might prevent the theoretical increases of accuracy and economy from being achieved. We know from published results that the mean square error in the absolute values of coordinates derived from machine coordinates measured in a single model with a first-order machine, compared with the known ground coordinates, is of the order of 12 to 15 μ in the plane of the negative. Using glass plate photography, ultra-flat plates, signalised points, compensation plates, air conditioning etc., the Austrian organisation already referred to above has reduced this figure to 10 μ. The accidental errors of instrumental measurement of a machine such as the A7 derived from grid measurements are about 5–6 μ so that the reduction of the instrumental accidental errors to 2–3 μ in a stereocomparator does not mean too much compared with the large remaining errors which are obviously due to other causes.

The principal one would appear to be that of the photography itself. The cameras produced to-day are very refined instruments with efficient shutters, and low distortion lenses, whose errors can be determined by calibration. The film is however the weak link. It possesses a flexible base, which is subjected to various stresses and strains in the camera, to widely varying temperatures and humidities and to the action of strong chemicals during processing. Distortions must occur, both regular and irregular, and there is the question of lack of film flatness at the moment of exposure. To use an instrument which can measure coordinates to 2 or 3 μ on such raw material would appear to be pointless, and there is little hope for increased accuracy in aerial triangulation until the inaccuracies and anomalies of the film itself are overcome.

There are methods of doing this, but they are not acceptable solutions to many. The first is the use of glass plate photography which has all the disadvantages of bulk and weight of the equipment against it. The second solution would be to incorporate a réseau in the film camera. The advantages of the réseau are well known—it will allow the complete determination and compensation of both regular and irregular film shrinkage and of lack of film flatness since changes over the 5 mm. between réseau crosses are likely to be negligible. In conjunction with a good camera calibration, it allows the elimination of nearly all errors due to the camera–film combination.

As a by-product, the use of the réseau also allows the simplification of the problem of measuring to 2 or 3 μ with a stereocomparator. To achieve this accuracy, great attention and care must be paid to the manufacture of the measuring spindles and to friction of the spindle which results in temperature changes if wound at high speed and also in changes due to wear. According to Schwidefsky[8] a distance of 20 mm. marked out on glass and steel will vary 1 μ per degree-change in temperature, which gives some idea of the problems involved.

To overcome these problems, the stereocomparator, even if in principle only a simple machine in comparison with the usual first-order machine, becomes a very expensive one. The Wild and Nistri stereocomparators which give measurements to 2 or 3 μ over the whole surface of the photograph approach in cost the equivalent first-order machines, so that the hoped-for saving in cost by the introduction of the stereocomparator has not been realised. This is especially important, particularly for military, commercial and the smaller government organisations, which will still, I think, prefer to spend the same money on the equivalent first-order machine which can perform aerial triangulation, numerical surveys for cadastral or revision purposes or plot graphically a map all to a very high precision indeed. The cost of these large accurate stereocomparators would appear to restrict their use straight away to very large organisations which have a continual demand for aerial triangulation, say for control for the standard topographic mapping of a large area.

The use of the réseau means that the cost of the comparator is much reduced, as the accuracy of measurement has only to be kept over the distance between crosses. The new Watts and Zeiss stereocomparators, which in different ways measure over a small distance, either using a reference grid plate or réseau crosses, are therefore simpler and much cheaper instruments, and would appear to be more attractive to many organisations where instrumental costs are a major consideration.

So the use of the réseau has great advantages both from the point of view of accuracy (as it reduces the errors of the film to negligible amounts) and of economy (because of simpler instrumentation), and many of the theoretical advances and benefits to be got from the introduction of the analytical method do not seem possible without it. What are the disadvantages? On the theoretical side there are none, but on the practical side there is the disadvantage that only a few réseau cameras are available, and these do not possess the superior illumination, high resolving power, low radial and asymmetrical distortion, and high shutter speeds of the currently popular Wild and Zeiss film cameras.

There is one serious factor that there is no solution for and this is random atmospheric refraction. The normal refraction correction is large—Tewinkel[7] has quoted that at an altitude of 20,000 feet for an image 40 degrees from the vertical axis it is three times the magnitude of the maximum distortion for a modern "distortion-free" lens. Such a systematic effect can be compensated for by computation in the analytical method or by incorporating the correction when manufacturing the appropriate correction plates or cams. However, refraction is continually changing and the photographic image has been taken during some unknown phase of the refraction taking place between the object and the camera. Since this effect is unknown in quantity and impossible to determine, it may place a limit on the ultimate accuracy which may be got by photogrammetric methods of providing control whether analytical or otherwise.

Listed in the advantages of analytical aerial triangulation was the possibility of simultaneously solving and adjusting by least squares a complete strip or block of photographs to all given ground control using electronic computers. Before the advent of this equipment, the adjustment of aerial triangulation was carried out using one of the various graphical interpolation methods of strip adjustment, especially those of the Swiss school, Zarzycki and Brandenberger. The least-squares adjustment of a strip to ground control has been formulated by Roelofs[8] but in practice such a method is too complicated and time consuming for production use. Discrepancies between the coordinates of common points derived from adjacent strips are treated by the succeeding block adjustment. In many cases this seems to involve simply averaging the conflicting answers and then adjusting the mean values from a graph constructed by comparison of computed and known coordinates of control points.

No block adjustment by least squares using all the relevant information could normally be attempted for even a small block, as this involves the solution of several thousand equations. The use of a correct least-squares block adjustment will certainly improve both the relative accuracy (i.e. the discrepancies between coordinates of a single point measured in different models) and the absolute accuracy (the discrepancies between the coordinates computed from the aerial triangulation and the known coordinates). This results not only in greater accuracies, but in the need for less ground control than is required for adjustment by the current methods. Straight away a saving is produced without any new instruments if this correct treatment can be applied.

Therefore, of the greatest importance to a discussion of the possible reduction or replacement of ground control by the application of photogrammetric methods is the development of the new International Training Centre–Jerie method of block adjustment. This method gives *mechanically* a least-squares adjustment of a block of aerial triangulation. It is in fact a mechanical analogue computer utilising stereo-templets which are connected elastically. The various contradictions are introduced mechanically at a much larger scale than the templets, and the block then takes up a position of equilibrium, when the potential energy is a minimum. The energy used in attaining this is proportional to the square of the displacements which took place, and the sum of the squares of the displacements must be a minimum which corresponds to the demands of the method of least squares. A full description of the working of the Jerie analogue computer would need a complete paper in itself but this outline will suffice for this discussion.†

This mechanical solution is of very great importance as the whole procedure can be carried out quickly by relatively unskilled staff. The necessary computations for the Jerie method are only an infinitesimally small amount of those necessary for a completely analytical solution of the same problem. The analytical solution is now economically possible on electronic computers, but the running costs of the large machines needed to solve the large numbers of simultaneous equations are very high; and very skilled personnel indeed are required for the formulation of the necessary equations and their subsequent programming on the computer. It is probable that the Jerie analogue method is not only much simpler but much cheaper, but this latter point must await an objective comparison. Certainly the Jerie analogue method allows the immediate and economic implementation of a least-squares solution to aerial triangulation, with the consequent economies which accrue.

In Lieut.-Colonel Irwin's paper already quoted[3] the absolute accuracy of the 30 check points used in the block (8 strips of 16 models each at 1/18,000 photo scale) which contained eight control points was $m_x = 1{\cdot}06$ metres; $m_y = 0{\cdot}73$ metres, giving a standard vector error of 1·29 metres. These results were arrived at after adjustment by the standard Ordnance Survey method described by Brigadier Shewell.[5] By using the Jerie analogue method and exactly the same basic material, the absolute accuracy was improved to $m_x = 0{\cdot}58$ metres; $m_y = 0{\cdot}52$ metres: giving a standard vector error of 0·78 metres, i.e. an almost twofold improvement in accuracy simply by being able to apply the optimum solution to the problem. The relative accuracy was also very good, the standard errors being $m_x = m_y = 0{\cdot}25$ metres, derived from examination of the discrepancies left at the tie points common to several models on completion of the adjustment. Such results, and similar ones have been achieved with the method in Holland and Austria, leave no doubt as to the benefits of this International Training Centre–Jerie block adjustment. Its immediate effects are the elimination of a great deal of the control necessary for aerial triangulation adjustment by other methods, without any deterioration in accuracy.

Having discussed the current and possible future methods and instruments for providing control by photogrammetric methods, it is instructive to see to what extent they are actually applied. This varies enormously according to the type of mapping required, and also from country to country even for the same type of job.

Small to Medium Scale Topographic Mapping

For small scale work (1/50,000, 1 inch to 1 mile, 1/100,000) photogrammetric control methods are already extensively applied with only a minimum of ground control points, fixed by astronomical means for the smallest scales and by long distance triangulation, trilateration or traversing for the larger ones. Provision of height control is normally carried out by trigonometric or barometric levelling.

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† A description has since appeared in *Photogrammetria.*

The actual method used depends on the requirements for precision, the urgency of the job and the instruments available. The value of aerial triangulation for this work is however widely recognised, where it would be quite uneconomical or impossible to provide ground control points.

Many organisations, particularly for mapping vast areas in underdeveloped countries abroad, have used slotted templet for planimetry and triangulation in a plotting machine such as the Multiplex for height. Presumably the use of stereo-templets can be expected to increase the accuracy of the planimetry, but it is quite certain that the use of full aerial triangulation with a first-order instrument, or the analytical method together with a correct block adjustment, will provide a very much more accurate solution, and reduce the number of necessary ground control points even further. This may well be not only the most accurate but perhaps also the cheapest solution, with these savings in control. If the extra control already exists, as sometimes occurs, or if there is only a need for limited precision and there are difficulties in acquiring capital for the purchase of equipment for full spatial aerial triangulation, then the first solution will probably continue to be used.

For small scale mapping, a 50 feet contour interval is quite common and the provision of control heights by the methods described above should be quite adequate. With flat terrain and a small contour interval (say 20 feet), photogrammetric methods will still be possible with perhaps denser control for heights, or a reduction in flying heights with its corresponding increases in cost.

Most of the current thought with regard to providing further economies in ground control seems to revolve round the use of smaller scale photographs, cutting down the number of overlaps to cover a given area without losing precision, particularly in height. Three different approaches to the problem are possible.

(*a*) The most obvious is the use of greater flying heights using the currently popular wide-angle cameras. Such a solution has been adopted in the Aden Protectorate where the previously existing 1/40,000 scale photographic cover was not an economic solution for the required 1/100,000 scale mapping. The photography was reflown at 45,000 feet using Canberra aircraft carrying a 6-inch camera, resulting in 1/90,000 photographs. The savings on the number of overlaps, and therefore in the number of photographic runs required and the number of orientations necessary in a plotting instrument, must be substantial, as well as the savings in the provision of control.

There are however some serious objections to this solution. First of all aircraft which can fly at such great altitudes are rarely available to commercial and government survey organisations, while the cost of purchasing and operating such equipment would probably be impossible to justify. In military organisations, however, where the necessary equipment already exists this will not apply.

Another objection is the difficulty of getting suitable weather conditions for such high altitude photography. Cloud is more likely to be a problem than at lower flying heights and there are more difficulties with haze. Nothing can be done about the former, but to help combat the haze problem and so allow greater flying heights, Wild have produced the Infragon wide-angle lens.

Generally this approach of utilising greater flying heights is one likely to appeal to military organisations which already possess or can quickly command the use of suitable equipment.

(*b*) Another approach, advocated particularly in the United States, is the use of twin convergent wide-angle photography with the vertical axis of each camera inclined 20 degrees from the vertical. Compared with vertical wide-angle photography flown at the same height the following advantages can be realised.

(i) The model area is almost doubled, with resulting savings in photography, ground control and orientations.

(ii) The base/height ratio is doubled from 0·6 for wide-angle verticals to 1·23, which results in an increase in the accuracy of height determination.

(iii) If accuracy requirements are the same, an increase in flying height is possible by using convergent wide-angle photography resulting in an even bigger increase in model area than stated in (i). The increase would be about 3·3 times the model area covered by the corresponding verticals.

The objections which can be brought against the adoption of twin convergent wide-angle photography are by no means minor ones. First of all there is the objection that contact prints are not suitable for mosaic and interpretation work due to the scale differences of the two corresponding photographs, a point which will carry greater or less weight according to the extent that an organisation wishes to utilise the photography for such purposes.

Another more important criticism is that the base/height ratio is in fact too high, resulting in difficulties with stereoscopic vision and the tendency for the stereoscopic image to dissolve into separate pictures. With large angles of convergence (47 to 64 degrees) between corresponding rays there will also be many dead spots and this will mean that houses, trees, hills, etc., will not appear as solid objects.

Many of these objections to the American convergent system have been raised by Dr. Brucklacher of Zeiss. His work[9] has shown that the increase in accuracy of height measurements is not a straight function of the increase in base/height ratio but falls off a great deal with ratios greater than 1·00. In a recent article[10] he has suggested a reduction of the inclination of the individual cameras to 13·5 degrees so reducing the base/height ratio to about 0·95, and reducing in turn some of the difficulties with stereoscopic vision and dead areas, without a loss in economy compared with the 20 degrees system. These criticisms and suggestions coming as they do from an organisation which advocates convergent photography for large scale work, whose Stereoplanigraphs can handle convergent photographs without alteration and whose existing wide-angle cameras can easily be used for any angle of convergency, must be given some weight.

From the point of view of radial triangulation, e.g. slotted templet triangulation, there is no objection to convergent photography which can be dealt with in a more sophisticated slotter such as the Zeiss R.S. I. Conventional spatial aerial triangulation with convergent photography is more difficult. The inner orientation of the photographs must be re-established to a high degree of accuracy to allow the theoretical increases of precision to be attained, while the mounts for a system such as the Twinplex would have to be very exactly calibrated. Doubts on these scores must however remain doubts, as published results of practical work on aerial triangulation with wide-angle convergent photography are conspicuously absent. Another objection which applies to both aerial triangulation and plotting is the inability to use the numerous existing mechanical projection machines for the work. Both Santoni and Wild have produced special versions of existing instruments to overcome such objections, but these alterations applied to existing instruments are likely to be expensive and therefore unattractive to their owners. Difficulties with instruments are less likely to be relevant in the United States where Government organisations use the Stereoplanigraph extensively, and the standard projection plotters such as the Balplex and Kelsh are easily adapted to use convergent photography.

(*c*) The third new approach to small scale work is the use of super-wide-angle photography, developed initially in the U.S.S.R. and now capable of adoption in the West with the development of suitable cameras and plotting instruments by Wild. The advantages of using super-wide-angle photography can be summarised as follows:—

(i) With the larger angular coverage of the super-wide-angle lens, each photograph covers an area two and a half times as great as that covered by a wide-angle lens. This results in a reduction in the number of overlaps and the number of photographic runs needed to give photographic cover of the area to be mapped. A reduction in the number of control points follows.

(ii) With the increase in base/height ratio to 1·0, there is a corresponding increase in the accuracy of height determination. If the accuracy requirements are the same an increase in the flying height is possible allowing even greater economies compared with the use of wide-angle photographs.

These advantages are similar to those to be gained by the use of twin convergent wide-angle photographs, though without all the difficulties of stereoscopic observation brought about by the use of 20 degrees off vertical convergent cameras. Some difficulties with stereoscopy and dead areas can be expected, however, and, to overcome them, Wild recommend the use of 80 per cent overlap during photography. Aerial triangulation is still possible using every third photograph (60 per cent overlap) and a base/height ratio of 1·0 with the advantages in accuracy and economy, while the 80 per cent overlaps, which still will have a base/height ratio of 0·5, can be used for plotting with fewer dead areas.

Super-wide-angle photography can of course be used for mosaicing and interpretation work without rectification. The criticism of convergent wide-angle photography on the grounds of lack of instruments applies perhaps even more to super-wide-angle photography. No instrument in current use apart from the Wild A9 Autograph can deal with super-wide-angle photography whereas an organisation already possessing wide-angle cameras and optical projection plotting instruments can introduce convergent photography with small modifications to existing equipment. Super-wide-angle photography has perhaps some advantages over convergent wide-angle photography, but its introduction does entail the purchase of new cameras and plotting instruments.

It seems probable that all three possibilities (*a*), (*b*), and (*c*) will be applied in future with the very considerable economies in ground control which result, together with the possibilities of greater accuracy which will be derived from the improvements in instruments and techniques previously described. I think we may expect that ground control will be reduced to the barest minimum for small scale mapping, perhaps 8 points with full control for a block of 8 strips of 15 to 20 photographs, with some extra points for height; but the required precision will be the final deciding factor. It should be remembered that such a block of super-wide-angle photography would cover an enormous expanse of ground.

These developments in techniques and instruments will in the near future also allow photogrammetric methods of providing control similarly to replace ground control by traversing and triangulation for all medium scale mapping down to 1/10,000 scale. The actual extent to which aerial triangulation can be used is dependent on the specification and on the type of country which has to be mapped. If we have very flat ground, and a contour interval of 5 ft. specified, then the very best techniques and instruments would have to be employed to use full aerial triangulation with only a few overlaps between ground control points, the difficulty being to meet the required precision in height. The use of analytical radial triangulation to give the required planimetric coordinates while supplying control for height by a quick method of ground levelling (e.g. use of tacheometry or self-levelling levels such as the Zeiss Ni2) might be more economic than full aerial triangulation in flat terrain.

With larger contour intervals, aerial triangulation for medium scale mapping must surely become standard practice, the length of strip and therefore the density of ground control being determined by the precision required in height and the equipment available to the user. A publication by the Ordnance Survey on the results achieved using analytical aerial triangulation for the supply of control for the 6-inch (1/10,560) resurvey of the Scottish Highlands would be of considerable value and interest as, unlike the previous work done by the Ordnance Survey at larger scales, heights are supplied as well as the planimetric coordinates.

Large Scale Mapping

At scales larger than 1/5000, it is more difficult to see ground methods of supplying control being superseded by photogrammetric methods, certainly not to the extent which can be expected in medium and small scale mapping. In this connexion, systematic mapping of a country at a large scale such as the 1/2500 Ordnance Survey series is simply unheard of outside the United Kingdom. Large scale mapping abroad is only carried out for small, intensely developed areas, for cadastral purposes, and for engineering projects. Photogrammetry in such cases cannot be applied to intensive, high precision work with quite the same economy as in extensive small scale work, which would be utterly impossible without it. This is not to say in any way that photogrammetry is not applicable to large scale work, but rather to point out that the circumstances are completely different and often not so favourable to the provision of control. Plotting of single models, or, as we have seen, the provision of numerical coordinates for cadastral and revision points, are well within the capacity of present first-order plotting machines. The enormous saving in control merely by adopting photogrammetric plotting instead of detail survey on the ground has already been spoken of on page 128. Dr. Harry has given further information on this point.[11] For cadastral work in Switzerland at large scales, 200 points per sq. km. were necessary for control using wholly ground methods as against 10 points per sq. km. for photogrammetric surveys. However, the very high precisions asked for, particularly for height, e.g. in engineering surveys, and also in planimetry, e.g. in numerical cadastral work, are such that aerial triangulation cannot be applied in the extensive way possible in small and medium scale work. It can be applied in these circumstances, but only over a few models, or the requirements for precision of the control points will not be met. So there is still quite a large amount of control needed to be fixed by the traditional ground methods such as traversing.

For road surveys, the final plan is often supplied at scales from 1/600 to 1/2000, a very frequent task for the commercial air survey companies in Britain nowadays. A single long strip of photographs is flown, and no doubt satisfactory planimetric accuracy can be achieved by aerial triangulation, say with ground control every fifth or sixth model. When however the specification calls for 2-foot to 5-foot contours, and it is necessary to know the values of height control points to perhaps 4 to 9 inches, the necessary precision cannot be met by bridging.

Since it is also necessary to establish numerous stations in the terrain for later setting-out work, and since in any case ground traversing and spirit levelling will be carried out along the line of the road for control purposes, the provision of extra points for full ground control will incur very little extra effort or expense.

It is of course possible, with the increases of accuracy to be expected from the new methods and instruments already discussed, that aerial triangulation may be applied at large scales in favourable circumstances such as, e.g., the 1/2500 Ordnance Survey series where the height control is supplied from the dense levelling net already existing on the ground. In less favourable circumstances terrestrial methods of supplying control must continue to be used to a considerable extent.

Non-technical Considerations

All these considerations with regard to the provision of control by photogrammetric methods are perhaps taken into account by many photogrammetrists, but it is quite certain that, even with present-day methods and instruments, aerial triangulation is not applied to anything like the extent it could be; and furthermore, when it is applied, it often is applied in a way which is far from the best solution. It is worth discussing some of the possible reasons for this situation as they will almost certainly be just as relevant in the future.

The first reason is possibly the lack of capital to invest in such expensive equipment as first-order plotting instruments, stereocomparators, electronic computers and the like. No doubt the application of these instruments would result in large economies in the long run, but the initial investment is heavy, and beyond the resources of smaller commercial companies and even government organisations whose requests for money have to compete with the thousands of more urgent demands made on a Finance Department by other government agencies. The existing equipment for ground survey or for aerial triangulation by less efficient instruments may be still serviceable, so that the Finance Department, although perhaps sympathetic and appreciative of the possible savings to be got by the adoption of new methods, turns down requests for money for new, very expensive equipment.

The presence of several fairly new Multiplex bars in an office means that the photography is flown to suit the instruments and the ground control placed to suit the number of projectors or the probable accuracy of bridging with the equipment. Again, other organisations may have invested heavily in plotting instruments utilising the Porro-Koppe principle, and they may find it difficult to change over to using modern distortion-free high quality aerial cameras which would allow greater flying heights, or let us say, to super-wide-angle cameras, because of the heavy cost of changing all the plotting cameras. Many other examples of considerations such as these can be quoted, and no doubt most people are familiar with this very important aspect of the whole question of providing control by photogrammetric methods.

The second reason for lack of application of the method would appear to be the lack of knowledge or lack of training of photogrammetric personnel. Some of the heads of survey organisations and departments are very experienced and competent ground surveyors, who are not familiar with all the possible advantages of photogrammetric techniques and in extreme cases are even hostile to their too extensive application. Cases are still found where all the necessary control points are determined by the traditional ground survey methods rather than by aerial triangulation, which needs well trained personnel for its execution and, in particular, for the computations and adjustment. This is an important point for the future, as very competent and highly trained photogrammetrists will be needed for the introduction of techniques such as analytical aerial triangulation, and yet only in a few institutes in the world can anything approaching an adequate education in advanced photogrammetry be found. The two problems of capital and personnel may be just as important factors in deciding how far photogrammetric methods of providing control are applied in the future as any of the more technical considerations previously discussed.

Finally, although photogrammetric methods would appear almost certain to displace the traditional ground methods more and more on the grounds of economy, it is worth keeping in mind the revolution in ground methods themselves which will certainly take place in the near future. Over the last 40 years there have been few really big changes in ground survey technique—equipment has gradually become lighter, easier to handle and more accurate and methods a little more refined. However, the invention and perfection over the last 5 years of new instruments such as electronic distance measuring equipment, precision barometers, and self-levelling levels will result in radical changes in techniques and very large economies in ground control costs.

Ground triangulation will surely in many cases be replaced by trilateration and precision traverses using very portable, accurate instruments such as the Tellurometer and the newer (models 3 and 4) Geodimeters. The use of these instruments in conjunction with helicopters in more difficult terrain must produce great savings in time and money. The precise traverse in particular would seem now to have a big future as it can be used without the elaborate reconnaissance necessary to obtain well-conditioned triangles to carry forward accurate triangulation. The astonishing first-order traverse in Kenya reported by Lieut.-Colonel Humphries and H. Brazier[12] is a case in point. Using the Tellurometer, a 403-mile traverse with 26 legs across a very flat area was measured in 28 days, against the estimated 2 to 2½ years using the old method with precision taping.

For height control, the development of precision microbarographs and the self-levelling level must be important for small scale and very large scale work respectively, and considerable economies can be expected from their introduction. Whether all these new developments might mean that ground methods could begin to challenge photogrammetric methods as the more economical means of supplying control for mapping seems doubtful. It would seem rather that what ground control is still needed for mapping will be produced in a quicker and more economical way. Now that photogrammetry has been proved to give the accuracies for control work previously only possible on the ground, its economy is unrivalled especially for small and medium scale work. The traditional methods of ground survey will continue however to play a considerable part in controlling large scale work.

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